Partial and total correctness

Our explanation of when the triple $(\!|\phi|\!)\ P\ (\!|\psi|\!)$ holds was rather informal. In particular, it did not say what we should conclude if P does not terminate. In fact there are two ways of handling this situation. *Partial correctness* means that we do not require the program to terminate, whereas in *total correctness* we insist upon its termination.

Definition 4.5 (Partial correctness) We say that the triple $(\!|\phi|\!)\ P\ (\!|\psi|\!)$ is satisfied under partial correctness if, for all states which satisfy ϕ, the state resulting from P's execution satisfies the postcondition ψ, provided that P actually terminates. In this case, the relation $\vDash_{\mathsf{par}} (\!|\phi|\!)\ P\ (\!|\psi|\!)$ holds. We call $\vDash_{\mathsf{par}}$ the satisfaction relation for partial correctness.

Thus, we insist on ψ being true of the resulting state only if the program P has terminated on an input satisfying ϕ. Partial correctness is rather a weak requirement, since any program which does not terminate at all satisfies its specification. In particular, the program

```
while true { x = 0; }
```

– which endlessly 'loops' and never terminates – satisfies all specifications, since partial correctness only says what must happen *if* the program terminates.

Total correctness, on the other hand, requires that the program terminates in order for it to satisfy a specification.